Progressions Questions for XAT PDF

Instructions
For the following questions answer them individually

Question 1

The sum of 3rd and 15th elements of an arithmetic progression is equal to the sum of 6th, 11th and 13th elements of the same progression. Then which element of the series should necessarily be equal to zero?

A 1st

B 9th

C 12th

D None of the above

Answer: C

Explanation:
The sum of the 3rd and 15th terms is a+2d+a+14d = 2a+16d
The sum of the 6th, 11th and 13th terms is a+5d+a+10d+a+12d = 3a+27d
Since the two are equal, 2a+16d = 3a+27d => a+11d = 0
So, the 12th term is 0

Question 2

If the sum of the first 11 terms of an arithmetic progression equals that of the first 19 terms, then what is the sum of the first 30 terms?

A 0

B -1

C 1

D Not unique

Answer: A

Explanation:
Sum of the first 11 terms = 11/2 (2a+10d)

Sum of the first 19 terms = 19/2 (2a+18d)

=> 22a+110d = 38a+342d => 16a = -232d

=> 2a = -232/8 d = -29d

Sum of the first 30 terms = 15(2a+29d) = 0

Question 3

a, b, c, d and e are integers such that 1 ≤ a < b < c < d < e. If a, b, c, d and e are geometric progression and lcm (m , n) is the least common multiple of m and n, then the maximum value of $\frac{1}{lcm(a,b)} + \frac{1}{lcm(b,c)} + \frac{1}{lcm(c,d)} + \frac{1}{lcm(d,e)}$ **is**

A 1

B 15/16

C 78/81

D 7/8

E None of these

Answer: B

Explanation:
Given that the numbers are in G.P.

Let the common ratio be 'r', hence the series a,b,c,d,e can also be expressed as:

a, ar, ar^2, ar^3, ar^4

lcm(a,b) = lcm$(a, ar) = ar$

lcm(b,c) = lcm$(ar, ar^2) = ar^2$

lcm(c,d) = lcm$(ar^2, ar^3) = ar^3$

lcm(d,e) = lcm$(ar^3, ar^4) = ar^4$

$\therefore \frac{1}{lcm(a,b)} + \frac{1}{lcm(b,c)} + \frac{1}{lcm(c,d)} + \frac{1}{lcm(d,e)}$

$= \frac{1}{ar} + \frac{1}{ar^2} + \frac{1}{ar^3} + \frac{1}{ar^4}$

$= \frac{1}{a}(\frac{1}{r} + \frac{1}{r^2} + \frac{1}{r^3} + \frac{1}{r^4})$

To get max value of this, 'a' and 'r' should be minimum.

It is given that $1 \leq a$ => Minimum value of 'a' = 1

For the values in the series to be integers, the minimum common ratio, r = 2 ($r \leq 1$ won't work here as it is an increasing GP)

Substituting values of 'a' and 'r' in the expression, we get :

Max value = $\frac{1}{1}(\frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \frac{1}{2^4})$

$= \frac{8+4+2+1}{16} = \frac{15}{16}$

XAT Previous Papers

Question 4

If 2, a, b, c, d, e, f and 65 form an arithmetic progression, find out the value of 'e'.

A 48

B 47

C 41

D None of the above

Answer: B

Explanation:
Given that 2, a, b, c, d, e, f and 65 are in an AP.

65 = 2 + (8-1)d

d = 9.

Therefore, e = 2+(6-1)*9 = 2+45 = 47. Therefore, option B is the correct answer.

Question 5

Suppose a, b and c are in Arithmetic Progression and a^2, b^2 and c^2 are in Geometric Progression. If $a < b < c$ and a+b+c=$\frac{3}{2}$,, then the value of a=

A $\frac{1}{2\sqrt{2}}$

B $\frac{1}{2\sqrt{3}}$

C $\frac{1}{2} - \frac{1}{\sqrt{3}}$

D $\frac{1}{2} - \frac{1}{\sqrt{2}}$

Answer: D

Explanation:
Let us assume that the common difference of the A.P. is 'd'.

Then, we can say that a = b - d, c = b + d

It is given that a + b + c = 3/2. i.e. b = 1/2.

It is given that a^2, b^2 and c^2 are in Geometric Progression. Hence, we can say that

$b^4 = a^2 * c^2$

$b^4 = (b-d)^2 * (b+d)^2$

$b^4 = (b^2 - d^2)^2$

$\Rightarrow (b^2 + d^2 - b^2)(b^2 - d^2 + b^2) = 0$

Therefore, $(b^2 - d^2 + b^2) = 0$. i.e. $d = \frac{1}{\sqrt{2}}$

Hence, a = b - d = $\frac{1}{2}$ - $\frac{1}{\sqrt{2}}$.

Question 6

If three positive real numbers a, b and c (c > a) are in Harmonic Progression, then log (a + c) + log (a - 2b + c) is equal to:

A 2 log (c - b)

B 2 log (a - c)

C 2 log (c - a)

D log a + log b + log c

Answer: C

Explanation:
It has been given that the terms $a, b,$ and c are in harmonic progression.
Therefore, $\frac{1}{b} - \frac{1}{a} = \frac{1}{c} - \frac{1}{b}$
$\frac{2}{b} = \frac{1}{a} + \frac{1}{c}$
$\frac{2}{b} = \frac{a+c}{ac}$
$b = \frac{2ac}{(a+c)}$--------------(1)
The given expression is log $(a+c)$ + log $(a-2b+c)$.
log $(a+c) + log(a-2b+c)$ = log $((a+c)(a-2b+c))$
Substituting (1), we get,

log $(a+c)$ + log $(a-2b+c)$ = log$((a+c)(a-\frac{4ac}{(a+c)}+c))$
= log $(a^2+ac-4ac+c^2+ac)$
= log (a^2+c^2-2ac)
= log $(c-a)^2$ [Since c is greater than a]
= 2 log $(c-a)$
Therefore, option C is the right answer.

XAT Free Mock Test

Question 7

The interior angles of a polygon are in Arithmetic Progression. If the smallest angle is 120° and common difference is 5°, then number of sides in the polygon is:

A 7

B 8

C 9

D None of the above

Answer: C

Explanation:
It has been given that the interior angles in a polygon are in an arithmetic progression.
We know that the sum of all exterior angles of a polygon is 360°.
Exterior angle = 180° - interior angle.
Since we are subtracting the interior angles from a constant, the exterior angles will also be in an AP.
The starting term of the AP formed by the exterior angles will be 180°-120° = 60° and the common difference will be -5°.

Let the number of sides in the polygon be 'n'.
=> The number of terms in the series will also be 'n'.

We know that the sum of an AP is equal to 0.5*n*(2a + (n-1)d), where 'a' is the starting term and 'd' is the common difference.
0.5*n*(2*60° + (n-1)*(-5°)) = 360°
120n - 5n^2 + 5n = 720
5n^2 - 125n + 720 = 0
n^2 - 25n + 144 =0.
$(n-9)(n-16)=0$

Therefore, n can be 9 or 16.
If the number of sides is 16, then the largest external angle will be 60 - 15*5 = -15°. Therefore, we can eliminate this case.
The number of sides in the polygon must be 9. Therefore, option C is the right answer.

Question 8

If the positive real numbers a, b and c are in Arithmetic Progression, such that abc = 4, then minimum possible value of b is:

A $2^{\frac{3}{2}}$

B $2^{\frac{2}{3}}$

C $2^{\frac{1}{3}}$

D None of the above

Answer: B

Explanation:
It has been given that a, b, and c are in an arithmetic progression.
Let a = x-p, b = x, and c = x+p
We know that a, b, and c are real numbers.
Therefore, the arithmetic mean of a,b,c should be greater than or equal to the geometric mean.
$\frac{a+b+c}{3} \geq \sqrt[3]{abc}$
$\frac{a+b+c}{3} \geq \sqrt[3]{4}$
$\frac{3x}{3} \geq \sqrt[3]{4}$
$x \geq \sqrt[3]{4}$
We know that x = b.
Therefore,$b \geq \sqrt[3]{4}$or $b \geq 2^{\frac{2}{3}}$
Therefore, option B is the right answer.

Question 9

If the square of the 7th term of an arithmetic progression with positive common difference equals the product of the 3rd and 17th terms, then the ratio of the first term to the common difference is

A 2:3

B 3:2

C 3:4

D 4:3

Answer: A

Explanation:
The seventh term of an AP = a + 6d. Third term will be a + 2d and second term will be a + 16d. We are given that $(a+6d)^2 = (a+2d)(a+16d)$
=> a^2 + $36d^2$ + 12ad = $a^2 + 18ad + 32d^2$
=> $4d^2 = 6ad$
=> $d : a = 3 : 2$
We have been asked about a:d. Hence, it would be 2:3

XAT Decision Making Mock Tests

Question 10

Let a_1, a_2,............., a_{3n} be an arithmetic progression with a_1 = 3 and a_2 = 7. If a_1+ a_2 +...+ a_{3n}= 1830, then what is the smallest positive integer m such that m(a_1+ a_2 +...+ a_n) > 1830?

A 8

B 9

C 10

D 11

Answer: B

Explanation:
a_1 = 3 and a_2 = 7. Hence, the common difference of the AP is 4.
We have been given that the sum up to 3n terms of this AP is 1830. Hence, $1830 = \frac{m}{2}[2*3+(m-1)*4$

=> 1830*2 = m(6 + 4m - 4)
=> 3660 = 2m + 4m^2
=> $2m^2 + m - 1830 = 0$
=> (m - 30)(2m + 61) = 0
=> m = 30 or m = -61/2
Since m is the number of terms so m cannot be negative. Hence, must be 30
So, 3n = 30
n = 10
Sum of the first '10' terms of the given AP = 5*(6 + 9*4) = 42*5 = 210
m(a_1+ a_2 +...+ a_n) > 1830
=> 210m > 1830
=> m > 8.71
Hence, smallest integral value of 'm' is 9.