Algebra Questions for CMAT

Instructions

For the following questions answer them individually

Question 1

What is the coefficient of x^2 in the expansion of $\left(5 - \frac{x^2}{3}\right)^3$?

A   -25

B $-\frac{25}{3}$

C   25

D $-\frac{5}{3}$

Answer: A

Explanation:

$\left(5 - \frac{x^2}{3}\right)^3 = \left(5 - \frac{x^2}{3}\right)\left(5 - \frac{x^2}{3}\right)^2$

$= \left(5 - \frac{x^2}{3}\right)\left(25 + \frac{x^4}{9} - \frac{10x^2}{3}\right)$

$= 125 + \frac{5x^4}{9} - \frac{50x^2}{3} - \frac{25x^2}{3} - \frac{x^6}{27} + \frac{10x^4}{9}$

$= -\frac{x^6}{27} + \frac{15x^4}{9} - \frac{75x^2}{3} + 125$

$= -\frac{x^6}{27} + \frac{5x^4}{3} - 25x^2 + 125$

The coefficient of x^2 in the expansion = -25

Hence, the correct answer is Option A

Question 2

Given that $x^8 - 34x^4 + 1 = 0, x > 0$. What is the value of $\left(x^3 - x^{-3}\right)$?

A   14

B   12

C   18

D   16

Answer: A

Explanation:

$x^8 - 34x^4 + 1 = 0$
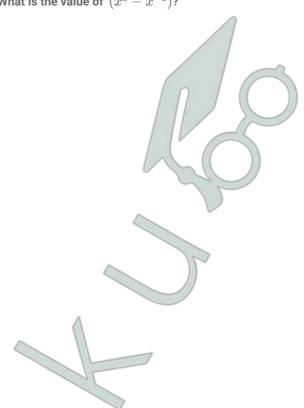
$x^8 + 1 = 34x^4$

$x^4 + \frac{1}{x^4} = 34$

$x^4 + \frac{1}{x^4} + 2 = 36$

$\left(x^2 + \frac{1}{x^2}\right)^2 = 36$

$x^2 + \frac{1}{x^2} = 6$

$x^2 + \frac{1}{x^2} - 2 = 4$

$\left(x - \frac{1}{x}\right)^2 = 4$

$x - \frac{1}{x} = 2$........(1)

$\left(x - \frac{1}{x}\right)^3 = 8$

$x^3 - \frac{1}{x^3} - 3.x.\frac{1}{x}\left(x - \frac{1}{x}\right) = 8$

$x^3 - \frac{1}{x^3} - 3(2) = 8$

$x^3 - \frac{1}{x^3} - 6 = 8$

$x^3 - \frac{1}{x^3} = 14$

Hence, the correct answer is Option A

Question 3

If $x^4 - 62x^2 + 1 = 0$, where $x > 0$, then the value of $x^3 + x^{-3}$ is:

A 500

B 512

C 488

D 364

Answer: C

Explanation:

$x^4 - 62x^2 + 1 = 0$

$x^4 + 1 = 62x^2$

$x^2 + \frac{1}{x^2} = 62$

$x^2 + \frac{1}{x^2} + 2 = 64$

$\left(x + \frac{1}{x}\right)^2 = 64$

$x + \frac{1}{x} = 8$........(1)

$\left(x + \frac{1}{x}\right)^3 = 512$

$x^3 + \frac{1}{x^3} + 3.x.\frac{1}{x}\left(x + \frac{1}{x}\right) = 512$

$x^3 + \frac{1}{x^3} + 3(8) = 512$

$x^3 + \frac{1}{x^3} + 24 = 512$

$x^3 + \frac{1}{x^3} = 488$

Hence, the correct answer is Option C

Question 4

If $x + \frac{1}{x} = \frac{17}{4}$, $x > 1$, then what is the value of $x - \frac{1}{x}$?

A $\frac{9}{4}$

B $\frac{3}{2}$

C $\frac{8}{3}$

D $\frac{15}{4}$

Answer: D

Explanation:

$x + \frac{1}{x} = \frac{17}{4}$

$\left(x + \frac{1}{x}\right)^2 = \frac{289}{16}$

$x^2 + \frac{1}{x^2} + 2 = \frac{289}{16}$

$x^2 + \frac{1}{x^2} = \frac{289}{16} - 2$

$x^2 + \frac{1}{x^2} = \frac{257}{16}$

$x^2 + \frac{1}{x^2} - 2 = \frac{257}{16} - 2$

$\left(x - \frac{1}{x}\right)^2 = \frac{257 - 32}{16}$

$\left(x - \frac{1}{x}\right)^2 = \frac{225}{16}$

$x - \frac{1}{x} = \frac{15}{4}$

Hence, the correct answer is Option D

Question 5

If $2x^2 - 7x + 5 = 0$, then what is the value of $x^3 + \frac{125}{8x^3}$?

A $12\frac{5}{8}$

B $16\frac{5}{8}$

C $10\frac{5}{8}$

D $18\frac{5}{8}$

Answer: B

Explanation:

$2x^2 - 7x + 5 = 0$

$2x^2 - 2x - 5x + 5 = 0$

$2x(x - 1) - 5(x - 1) = 0$

$(x - 1)(2x - 5) = 0$

$x - 1 = 0$ or $2x - 5 = 0$

$x = 1$ or $x = \frac{5}{2}$

When $x = 1$,

$x^3 + \frac{125}{8x^3} = (1)^3 + \frac{125}{8(1)^3} = 1 + \frac{125}{8} = \frac{133}{8} = 16\frac{5}{8}$

Hence, the correct answer is Option B

Question 6

If $x - \frac{1}{x} = 1$, then what is the value of $x^8 + \frac{1}{x^8}$?

A 3

B 119

C 47

D -1

Answer: C

Explanation:

$x - \frac{1}{x} = 1$

Squaring on both sides,

$x^2 + \frac{1}{x^2} - 2 = 1$

$x^2 + \frac{1}{x^2} = 3$

Squaring on both sides,

$x^4 + \frac{1}{x^4} + 2 = 9$
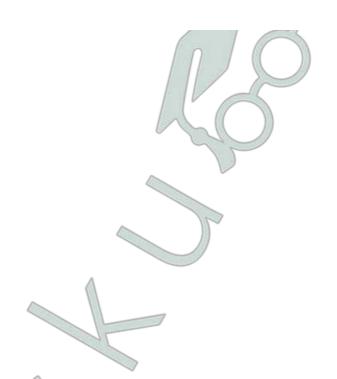
$x^4 + \frac{1}{x^4} = 7$

Squaring on both sides,

$x^8 + \frac{1}{x^8} + 2 = 49$

$x^8 + \frac{1}{x^8} = 47$

Hence, the correct answer is Option C

Take a CMAT free mock test

Question 7

If $x^4 + \frac{1}{x^4} = 727, x > 1$, then what is the value of $\left(x - \frac{1}{x}\right)$?

A 6

B -6

C -5

D 5

Answer: D

Explanation:

$x^4 + \frac{1}{x^4} = 727$

$x^4 + \frac{1}{x^4} + 2 = 729$

$\left(x^2 + \frac{1}{x^2}\right)^2 = 729$

$x^2 + \frac{1}{x^2} = 27$

$x^2 + \frac{1}{x^2} - 2 = 25$

$\left(x - \frac{1}{x}\right)^2 = 25$

Since $x > 1$,

$x - \frac{1}{x} = 5$

Hence, the correct answer is Option D

Question 8

If $2x^2 - 8x - 1 = 0$, then what is the value of $8x^3 - \frac{1}{x^3}$?

A 560

B 540

C 524

D 464

Answer: A

Explanation:

$2x^2 - 8x - 1 = 0$

$2x^2 - 1 = 8x$

$2x - \frac{1}{x} = 8$........(1)

Cubing on both sides,

$8x^3 - \frac{1}{x^3} - 3.2x.\frac{1}{x}\left(2x - \frac{1}{x}\right) = 512$

$8x^3 - \frac{1}{x^3} - 6\,(8) = 512$ [From (1)]

$8x^3 - \frac{1}{x^3} - 48 = 512$

$8x^3 - \frac{1}{x^3} = 560$

Hence, the correct answer is Option A

Question 9

If $y = 2x + 1$, then what is the value of $\left(8x^3 - y^3 + 6xy\right)$?

A 1

B -1

C 15

D -15

Answer: B
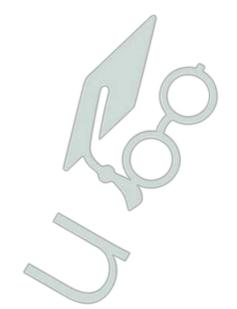
Explanation:

$y = 2x + 1$

$2x - y = -1$........(1)

Cubing on both sides, we get

$8x^3 - y^3 - 3.2x.y\,(2x - y) = -1$

$8x^3 - y^3 - 6xy\,(-1) = -1$ [From (1)]

$8x^3 - y^3 + 6xy = -1$

Hence, the correct answer is Option B

CMAT Free Solved Previous Papers.

Question 10

If $x - \frac{2}{x} = 15$, then what is the value of $\left(x^2 + \frac{4}{x^2}\right)$?

A 229

B 227

C 221

D 223

    Answer: A

Explanation:

$x - \dfrac{2}{x} = 15$

Squaring on both sides,

$x^2 + \dfrac{4}{x^2} - 2.x.\dfrac{2}{x} = 225$

$x^2 + \dfrac{4}{x^2} - 4 = 225$

$x^2 + \dfrac{4}{x^2} = 229$

Hence, the correct answer is Option A

Question 11

If $2x + 3y + 1 = 0$, then what is the value of $\left(8x^3 + 8 + 27y^3 - 18xy\right)$?

A -7

B 7

C -9

D 9

    Answer: B

Explanation:

$2x + 3y + 1 = 0$

$2x + 3y = -1$........(1)

Cubing on both sides,

$8x^3 + 27y^3 + 3.2x.3y\left(2x + 3y\right) = -1$

$8x^3 + 27y^3 + 18xy\left(-1\right) = -1$

$8x^3 + 27y^3 - 18xy + 8 = -1 + 8$

$8x^3 + 27y^3 - 18xy + 8 = 7$

Hence, the correct answer is Option B

Question 12

If $x + \dfrac{1}{x} = 7$, then $x^2 + \dfrac{1}{x^2}$ is equal to:

A 47

B 49

C 61

D 51

    Answer: A

Explanation:

$x + \dfrac{1}{x} = 7$

Squaring on both sides,

$x^2 + \dfrac{1}{x^2} + 2.x.\dfrac{1}{x} = 49$

$$x^2 + \frac{1}{x^2} + 2 = 49$$

$$x^2 + \frac{1}{x^2} = 47$$

Hence, the correct answer is Option A

Question 13

If $(2x + y)^3 - (x - 2y)^3 = (x + 3y)[Ax^2 + By^2 + Cxy]$, then what is the value of $(A + 2B + C)$?

A 13

B 14

C 7

D 10

Answer: D

Explanation:

$(2x + y)^3 - (x - 2y)^3 = (x + 3y)[Ax^2 + By^2 + Cxy]$

$[2x + y - (x - 2y)]\left[(2x + y)^2 + (2x + y)(x - 2y) + (x - 2y)^2\right] = (x + 3y)[Ax^2 + By^2 + Cxy]$

$[x + 3y]\left[4x^2 + y^2 + 4xy + 2x^2 - 3xy - 2y^2 + x^2 + 4y^2 - 4xy\right] = (x + 3y)[Ax^2 + By^2 + Cxy]$

$(x + 3y)\left[7x^2 + 3y^2 - 3xy\right] = (x + 3y)[Ax^2 + By^2 + Cxy]$

Comparing both sides,

A = 7, B = 3 and C = -3

$A + 2B + C = 7 + 2(3) - 3$ = 10

Hence, the correct answer is Option D

Question 14

If $9(a^2 + b^2) + c^2 + 20 = 12(a + 2b)$, then the value of $\sqrt{6a + 9b + 2c}$ is:

A 4

B 3

C 6

D 2

Answer: A

Explanation:

$9(a^2 + b^2) + c^2 + 20 = 12(a + 2b)$

$9a^2 + 9b^2 + c^2 + 20 = 12a + 24b$

$9a^2 - 12a + 9b^2 - 24b + c^2 + 20 = 0$

$9a^2 - 12a + 4 - 4 + 9b^2 - 24b + 16 - 16 + c^2 + 20 = 0$

$(3a - 2)^2 - 4 + (3b - 4)^2 - 16 + c^2 + 20 = 0$

$(3a - 2)^2 + (3b - 4)^2 + c^2 = 0$

$3a - 2 = 0,\ 3b - 4 = 0,\ c = 0$

$a = \frac{2}{3}, \, b = \frac{4}{3}, \, c = 0$

$\sqrt{6a + 9b + 2c} = \sqrt{6\left(\frac{2}{3}\right) + 9\left(\frac{4}{3}\right) + 2\,(0)}$

$= \sqrt{4 + 12}$

$= \sqrt{16}$

$= 4$

Hence, the correct answer is Option A

Question 15

If $x + \frac{1}{x} = 2\sqrt{5}$, then what is the value of $\frac{\left(x^4 + \frac{1}{x^2}\right)}{x^2 + 1}$?

A 14

B 17

C 20

D 23

Answer: B

Explanation:

$x + \frac{1}{x} = 2\sqrt{5}$..........(1)

$\left(x + \frac{1}{x}\right)^3 = 40\sqrt{5}$

$x^3 + \frac{1}{x^3} + 3.x.\frac{1}{x}\left(x + \frac{1}{x}\right) = 40\sqrt{5}$

$x^3 + \frac{1}{x^3} + 3\left(2\sqrt{5}\right) = 40\sqrt{5}$ [From (1)]

$x^3 + \frac{1}{x^3} + 6\sqrt{5} = 40\sqrt{5}$

$x^3 + \frac{1}{x^3} = 34\sqrt{5}$.........(2)

$\frac{\left(x^4 + \frac{1}{x^2}\right)}{x^2 + 1} = \frac{x\left(x^3 + \frac{1}{x^3}\right)}{x\left(x + \frac{1}{x}\right)}$

$= \frac{x^3 + \frac{1}{x^3}}{x + \frac{1}{x}}$

$= \frac{34\sqrt{5}}{2\sqrt{5}}$

$= 17$

Hence, the correct answer is Option B

Download Excellent App for CMAT Preparation

Take a CMAT free mock test

CMAT Free Solved Previous Papers.

CAT Previous Papers PDF