Tips, Formulae and Shortcuts for Remainder Theorem

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Cracku Tip 1 – Chinese Remainder Theorem

Chinese remainder theorem is useful when the divisor of any number is composite.

Let M be a number which is divided by a divisor N. The theorem states that if N is the divisor which can be expressed as $N = a*b$ where a and b are co-prime

Then,

$M \bmod N = ar_2x + br_1y$

Here $r_1 = M \bmod a$

And $r_2 = M \bmod b$

Here, $ax + by = 1$

Find the remainder when 344^{237} is divided by 119.
In the first look it looks difficult but if one knows the Chinese remainder theorem then question can be solved very easily.

$119 = 17*7$, So here a = 17 and b = 7

$344^{237} \bmod 17 = 4^{237} \bmod 17 = (4*16^{116}) \bmod 17 = 4*1 = 4$

Hence, we get $r_1 = 4$

Now, $344^{237} \bmod 7 = 1^{237} \bmod 7 = 1$, Hence, $r_2 = 1$

We know that $M \bmod N = ar_2x + br_1y$

Therefore, $344^{237} \bmod 119 = 17*1x + 7*4y = 17x + 28y$ (1)

We know that $17x + 7y = 1$

=> We can see that x = 5 and y = -12 satisfies the above equation.

Hence, putting the values of x and y in equation 1, we get
$344^{237} \bmod 119 = 17*5 - 28*12 = 85 - 336 = -251$
Converting this into positive remainder we get $357 - 251 = 106$

Hence, the required remainder is 106.

Chinese Remainder Theorem

Let's consider another example to understand it better
Find the remainder when 495^{2517} is divided by 78.

In this question also, the divisor is 78 which can be written as 13*6. So, we can use the Chinese remainder theorem in this question as well.

Let's take a = 13 and b = 6

So we can write 495^{2517} mod 78 = $13r_2x + 6r_1y$
=> $r_1 = 495^{2517}$ mod 13 = 1^{2517} mod 13 = 1
=> $r_2 = 495^{2517}$ mod 6 = 3^{2517} mod 6 = $(3^{2516}$ mod 2)*3 = 1*3 = 3

We also know that 13x + 6y = 1
x = 1 and y = -2 satisfies the above equation.

Hence, we can obtain the remainder as
495^{2517} mod 78 = $13r_2x + 6r_1y$ = 13*3*1 = 6*1*-2 = 39 – 12 = 27
Hence, the required answer is 27

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