Algebra Questions for CAT

Questions

Instructions
For the following questions answer them individually

Question 1

Suppose you have a currency, named Miso, in three denominations: 1 Miso, 10 Misos and 50 Misos. In how many ways can you pay a bill of 107 Misos?

A 17

B 16

C 18

D 15

E 19

Answer: C

Video Solution

Explanation:
If two 50 Misos are used, the 107 can be paid in only 1 way.

If one 50 Miso is used, the number of ways of paying 107 is 6 - zero 10 Miso, one 10 Miso and so on till five 10 Misos.

If no 50 Miso is used, the number of ways of paying 107 is 11 - zero 10 Miso, one 10 Miso and so on till ten 10 Misos.

So, the total number of ways is 18

Question 2

The price of Darjeeling tea (in rupees per kilogram) is 100 + 0.10n, on the nth day of 2007 (n=1, 2, ..., 100), and then remains constant. On the other hand, the price of Ooty tea (in rupees per kilogram) is 89 + 0.15n, on the nth day of 2007 (n = 1, 2, ..., 365). On which date in 2007 will the prices of these two varieties of tea be equal?

A May 21

B April 11

C May 20

D April 10

E June 30

Answer: C

Video Solution

Explanation:
Price of Darjeeling tea on 100th day= 100+(0.1*100)=110
Price of Ooty tea on nth day= 89+0.15n
Let us assume that the price of both varieties of tea would become equal on nth day where n<=100
So
89+0.15n=100+0.1n
n=220 which does not satisfy the condition of n<=100
So the price of two varieties would become equal after 100th day.
89+0.15n=110
n=140
140th day of 2007 is May 20 (Jan=31,Feb=28,March=31,April=30,May=20)

Question 3

When you reverse the digits of the number 13, the number increases by 18. How many other two-digit numbers increase by 18 when their digits are reversed?

A 5

B 6

C 7

D 8

E 10

Answer: B

Video Solution

Explanation:
Let the number be xy
10y + x = 10x + y + 18
=> 9y - 9x = 18
=> y - x = 2
So, y can take values from 9 to 4 (since 3 is already counted in 13)
Number of possible values = 6

Question 4

Three friends, returning from a movie, stopped to eat at a restaurant. After dinner, they paid their bill and noticed a bowl of mints at the front counter. Sita took one-third of the mints, but returned four because she had a momentary pang of guilt. Fatima then took one-fourth of what was left but returned three for similar reason. Eswari then took half of the remainder but threw two back into the bowl. The bowl had only 17 mints left when the raid was over. How many mints were originally in the bowl?

A 38

B 31

C 41

D None of these

Answer: D

Video Solution

Explanation:
Let the total number of mints in the bowl be n
Sita took n/3 - 4. Remaining = 2n/3 + 4
Fatim took 1/4(2n/3 + 4) - 3. Remaining = 3/4(2n/3 + 4) + 3
Eswari took 1/2(3/4(2n/3+4)+3) - 2
Remaining = 1/2(3/4(2n/3+4)+3) + 2 = 17
=> 3/4(2n/3+4)+3 = 30 => (2n/3+4) = 36 => n = 48
So, the answer is option d)

Question 5

At a certain fast food restaurant, Brian can buy 3 burgers, 7 shakes, and one order of fries for Rs. 120 exactly. At the same place it would cost Rs. 164.5 for 4 burgers, 10 shakes, and one order of fries. How much would it cost for an ordinary meal of one burger, one shake, and one order of fries?

A Rs. 31

B Rs. 41

C Rs. 21

D Cannot be determined

Answer: A

Video Solution

Explanation:
Let the price of 1 burger be x and the price of 1 shake be y and the prize of 1 french fries be z
3x + 7y + z = 120
4x + 10y + z = 164.5
=> x + 3y = 44.5
=> x = 44.5 - 3y
=> 3(44.5 - 3y) + 7y + z = 120 => z = 120 - 133.5 + 2y
So, x+y+z = 44.5 - 3y + y -13.5 + 2y = 31
So, the cost of a meal consisting of 1 burger, 1 shake and 1 french fries = Rs 31

Question 6

The number of solutions (x, y, z) to the equation $x - y - z = 25$, where x, y, and z are positive integers such that $x \leq 40, y \leq 12$, and $z \leq 12$ is

A 101

B 99

C 87

D 105

Answer: B

Video Solution

Explanation:
x - y - z = 25 and $x \leq 40, y \leq 12, z \leq 12$
If x = 40 then y + z = 15. Now since both y and z are natural numbers less than 12, so y can range from 3 to 12 giving us a total of 10 solutions.Similarly, if x = 39, then y + z = 14. Now y can range from 2 to 12 giving us a total of 11 solutions.
If x = 38, then y + z = 13. Now y can range from 1 to 12 giving us a total of 12 solutions.
If x = 37 then y + z = 12 which will give 11 solutions.
Similarly on proceeding in the same manner the number of solutions will be 10, 9, 8, 7 and so on till 1.
Hence, required number of solutions will be (1 + 2 + 3 + 4 . . . . + 12) + 10 + 11
= 12*13/2 + 21
78 + 21 = 99

Question 7

If x, y, z are distinct positive real numbers the $(x^2(y + z) + y^2(x + z) + z^2(x + y))/xyz$ would always be

A Less than 6

B greater than 8

C greater than 6

D Less than 8

Answer: C

Video Solution

Explanation:
For the given expression value of x,y,z are distinct positive integers . So the value of expression will always be greater than value when all the 3 variables are equal . substitute x=y=z we get minimum value of 6 .

$(x^2(y+z)+y^2(x+z)+z^2(x+y))/xyz$ = x/z + x/y + y/z + y/x + z/y + z/x

Applying AM greater than or equal to GM, we get minimum sum = 6

Question 8

What values of x satisfy $x^{2/3}+x^{1/3}-2<=0$**?**

A $-8 \leq x \leq 1$

B $-1 \leq x \leq 8$

C $1 \leq x \leq 8$

D $1 \leq x \leq 18$

E $-8 \leq x \leq 8$

Answer: A

Video Solution

Explanation:
Try to solve this type of questions using the options.

Subsitute 0 first => We ger -2 <=0, which is correct. Hence, 0 must be in the solution set.

Substitute 8 => 4 + 2 - 2 <=0 => 6 <= 0, which is false. Hence, 8 must not be in the solution set.

=> Option 1 is the answer.

Question 9

If pqr = 1, the value of the expression $1/(1+p+q^{-1})+1/(1+q+r^{-1})+1/(1+r+p^{-1})$

A p+q+r

B 1/(p+q+r)

C 1

D $p^{-1}+q^{-1}+r^{-1}$

Answer: C

Video Solution

Explanation:
Let p = q = r = 1

So, the value of the expression becomes 1/3 + 1/3 + 1/3 = 1

If we substitute these values, options a), b) and d) do not satisfy.

Option c) is the answer.

Question 10

The number of integers n satisfying -n+2 ≥ 0 and 2n ≥ 4 is

A 0

B 1

C 2

D 3

Answer: B

Video Solution

Explanation:
-n+2 >= 0
or n<=2
and 2n>=4
or n>=2
So we can take only one value of n i.e. 2

Question 11

Which of the following values of x do not satisfy the inequality $(x^2 - 3x + 2 > 0)$ **at all?**

A $1 \leq x \leq 2$

B $-1 \geq x \geq -2$

C $0 \leq x \leq 2$

D $0 \geq x \geq -2$

Answer: A

Video Solution

Explanation:
After solving given equation, we will have inequality resolved to:

(x-1)(x-2)>0

Or we can say range of x will be as follows:

x<1; x>2

Hence, option A has a set of values which don't lie in the possible range of x.

So the answer will be A.

Question 12

The number of positive integer valued pairs (x, y), satisfying 4x - 17 y = 1 and x < 1000 is:

A 59

B 57

C 55

D 58

Answer: A

Video Solution

Explanation:

y = $\frac{4x-1}{17}$

The integral values of x for which y is an integer are 13, 30, 47,.....

The values are in the form 17n + 13, where $n \geq 0$

17n + 13 < 1000

=> 17n < 987

=> n < 58.05

=> n can take values from 0 to 58 => Number of values = 59

Question 13

If | r - 6 | = 11 and | 2q - 12 | = 8, what is the minimum possible value of q / r?

A -2/5

B 2/17

C 10/17

D None of these

Answer: D

Video Solution

Explanation:

| r-6 | = 11 => r = -5 or 17

| 2q - 12 | = 8 => q = 10 or 2

So, the minimum possible value of q/r = 10/(-5) = -2

Question 14

If a and b are integers of opposite signs such that $(a+3)^2 : b^2 = 9 : 1$ **and** $(a-1)^2 : (b-1)^2 = 4 : 1$**, then the ratio** $a^2 : b^2$ **is**

A 9:4

B 81:4

C 1:4

D 25:4

Answer: D

Video Solution

Explanation:

Since the square root can be positive or negative we will get two cases for each of the equation.

For the first one,

a + 3 = 3b .. i

a + 3 = -3b ... ii

For the second one,

a - 1 = 2(b -1) ... iii

a - 1 = 2 (1 - b) ... iv

we have to solve i and iii, i and iv, ii and iii, ii and iv.

Solving i and iii,

a + 3 = 3b and a = 2b - 1, solving, we get a = 3 and b = 2, which is not what we want.

Solving i and iv

a + 3 = 3b and a = 3 - 2b, solving, we get b = 1.2, which is not possible.

Solving ii and iii

a + 3 = -3b and a = 2b - 1, solving, we get b = 0.4, which is not possible.

Solving ii and iv,

a + 3 = -3b and a = 3 - 2b, solving, we get a = 15 and b = -6 which is what we want.

Thus, $\frac{a^2}{b^2} = \frac{25}{4}$

Question 15

For how many integers n, will the inequality $(n-5)(n-10)-3(n-2) \leq 0$ be satisfied?

Answer:11

Video Solution

Explanation:

$(n-5)(n-10)-3(n-2) \leq 0$

=> $n^2-15n+50-3n+6 \leq 0$

=> $n^2-18n+56 \leq 0$

=> $(n-4)(n-14) \leq 0$

=> Thus, n can take values from 4 to 14. Hence, the required number of values are 14 - 4 + 1 = 11.

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Question 16

The minimum possible value of the sum of the squares of the roots of the equation $x^2+(a+3)x-(a+5)=0$ is

A 1

B 2

C 3

D 4

Answer: C

Video Solution

Explanation:

Let the roots of the equation $x^2+(a+3)x-(a+5)=0$ be equal to p, q

Hence, $p+q=-(a+3)$ and $p \times q=-(a+5)$

Therefore, $p^2 + q^2 = a^2 + 6a + 9 + 2a + 10 = a^2 + 8a + 19 = (a + 4)^2 + 3$

As $(a + 4)^2$ is always non negative, the least value of the sum of squares is 3

Question 17

A quadratic function f(x) attains a maximum of 3 at x = 1. The value of the function at x = 0 is 1. What is the value of f (x) at x = 10?

A -119

B -159

C -110

D -180

E -105

Answer: B

Video Solution

Explanation:

Let the function be $ax^2 + bx + c$.

We know that x=0 value is 1 so c=1.

So equation is $ax^2 + bx + 1$.

Now max value is 3 at x = 1.

So after substituting we get a + b = 2.

If f(x) attains a maximum at 'a' then the differential of f(x) at x=a, that is, f'(a)=0.

So in this question f'(1)=0

=> 2*(1)*a+b = 0

=> 2a+b = 0.

Solving the equations we get a=-2 and b=4.

$-2x^2 + 4x + 1$ is the equation and on substituting x=10, we get -159.

Question 18

If the roots of the equation $x^3 - ax^2 + bx - c = 0$ are three consecutive integers, then what is the smallest possible value of b?

[CAT 2008]

A $\frac{-1}{\sqrt{3}}$

B -1

C 0

D 1

E $\frac{1}{\sqrt{3}}$

Answer: B

Video Solution

Explanation:

b = sum of the roots taken 2 at a time.

Let the roots be n-1, n and n+1.

Therefore, $b = (n-1)n + n(n+1) + (n+1)(n-1) = n^2 - n + n^2 + n + n^2 - 1$
$b = 3n^2 - 1$. The smallest value is -1.

Question 19

Let p and q be the roots of the quadratic equation $x^2 - (\alpha - 2)x - \alpha - 1 = 0$. What is the minimum possible value of $p^2 + q^2$?

A 0

B 3

C 4

D 5

Answer: D

Video Solution

Explanation:

Let α be equal to k.

=> f(x) = $x^2 - (k-2)x - (k+1) = 0$

p and q are the roots

=> p+q = k-2 and pq = -1-k

We know that $(p+q)^2 = p^2 + q^2 + 2pq$

=> $(k-2)^2 = p^2 + q^2 + 2(-1-k)$

=> $p^2 + q^2 = k^2 + 4 - 4k + 2 + 2k$

=> $p^2 + q^2 = k^2 - 2k + 6$

This is in the form of a quadratic equation.

The coefficient of k^2 is positive. Therefore this equation has a minimum value.

We know that the minimum value occurs at x = $\frac{-b}{2a}$

Here a = 1, b = -2 and c = 6

=> Minimum value occurs at k = $\frac{2}{2}$ = 1

If we substitute k = 1 in $k^2 - 2k + 6$, we get 1 -2 + 6 = 5.

Hence 5 is the minimum value that $p^2 + q^2$ can attain.

Question 20

Ujakar and Keshab attempted to solve a quadratic equation. Ujakar made a mistake in writing down the constant term. He ended up with the roots (4, 3). Keshab made a mistake in writing down the coefficient of x. He got the roots as (3, 2). What will be the exact roots of the original quadratic equation?

A (6, 1)

B (-3, -4)

C (4, 3)

D (-4, -3)

Answer: A

Video Solution

Explanation:

We know that quadratic equation can be written as x^2-(sum of roots)*x+(product of the roots)=0.

Ujakar ended up with the roots (4, 3) so the equation is x^2-(7)*x+(12)=0 where the constant term is wrong.

Keshab got the roots as (3, 2) so the equation is x^2-(5)*x+(6)=0 where the coefficient of x is wrong .

So the correct equation is x^2-(7)*x+(6)=0. The roots of above equations are (6,1).