cracku

CAT Questions on Last Two Digits

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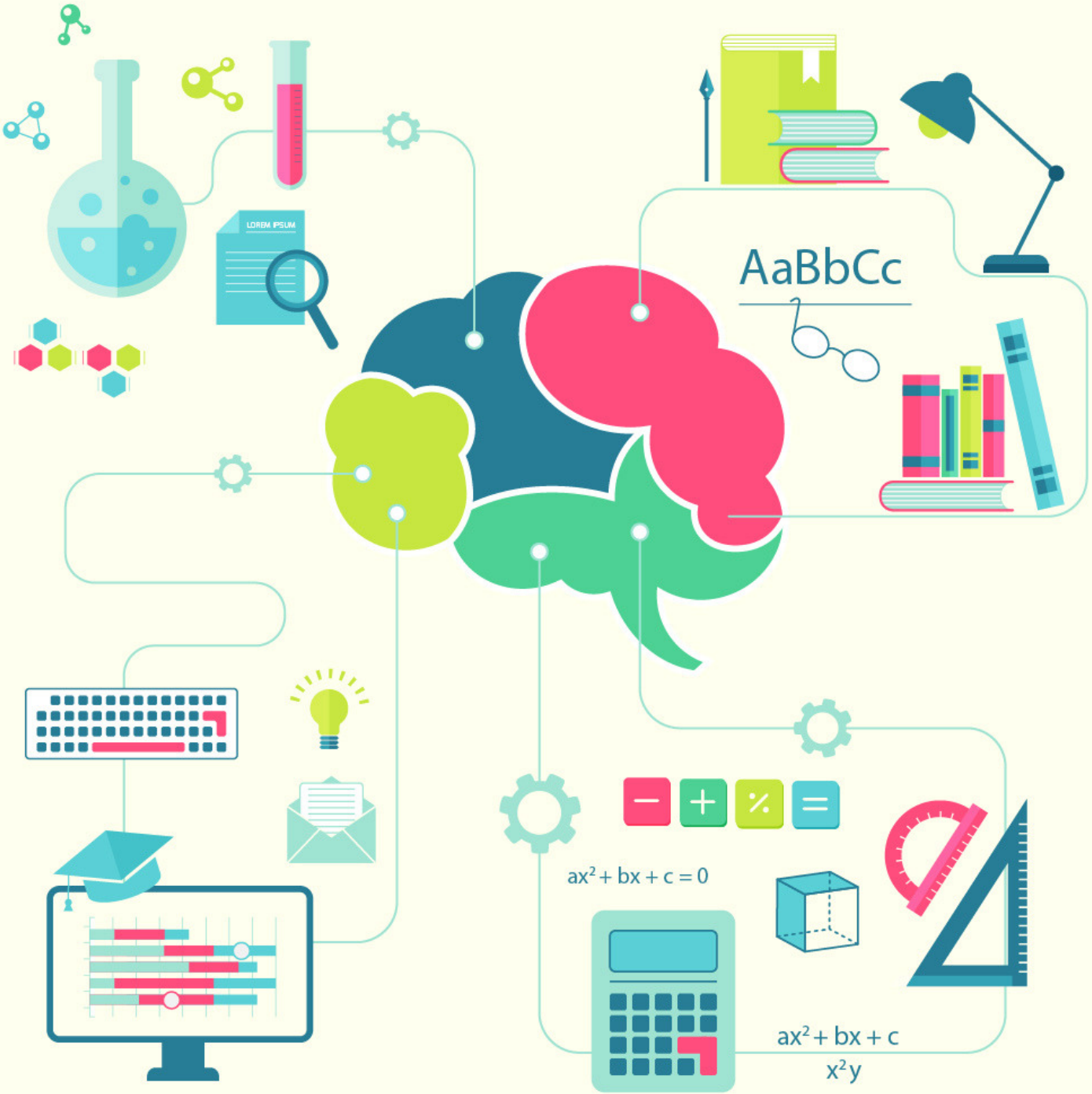

Question 1: If the last two digits of (1!+2!+3!..... 50!) are ab, then find the value of a*b?

a) 3
b) 4
c) 5
d) 6

Question 2: Find the last two non-zero digits of 25!.

a) 74
b) 94
c) 84
d) 64

Question 3: Find the sum of the last two digits in 89^{82}

a) 1
b) 3
c) 5
d) 7

Question 4: What are the last two digits of the sum $(19!)^2 + (18!)^2 + (17!)^2 + \cdots\cdots (0!)^2$

a) 33
b) 34
c) 17
d) 18

Question 5: Find the last two digits of the number 27^{103}

a) 63
b) 73
c) 93
d) 83

Solutions:

1) Answer (A)

Lat two digits is nothing but the remainder with 100
From 10! onwards all terms will be divisible by 100
So we just need to find out the remainder of (1! + 2! + ...9!) with 100
So on calculating the remainders we get,
1 + 2 + 6 + 24 + 20 + 20 + 40 + 20 + 80 = 213,
213mod 100 = 13, so last two digits are 13
Hence the product of digits is 1*3 = 3

2) Answer (C)

The last two non-zero digits of the factorial of a number X, which is divisible by 5 can be calculated as below.
last two non-zero digits of X! = last two non zero digits of $[\frac{x}{5}]! * 12^{x/5}$

Here x = 25 $\Longrightarrow [25/5]! * 12^{25/5} = 5! * 12^5 = 120 * 32(\textit{last two digits of } 12^5 \textit{is } 32) \Longrightarrow 84$
Therefore last two digits of 25! = 84.

3) Answer (B)

Last two digits can be found by finding the remainder with 100.
$89^{82} = (90 - 1)^{82}$
$= (...00) - (82 \times 90) + 1$
$= (...00) - 7380 + 1$
$= (...00) - 7379$
$= (...21)$
So the last two digits are 21.
Sum of digits = 3

4) Answer (D)

From $(5!)^2$ *to* $(19!)^2$ every number in the series ends with '00'.
Now we need to calculate the sum of last two digits of numbers from
$(0!)^2 to\ (4!)^2 = 1 + 1 + 4 + 36 + 576 = 618$
Therefore last two digits of the sequence are 18.

5) Answer (D)

Last two digits of a number is nothing but the remainder when the number is divided by 100. So we essentially have to find the remainder when the given number is divided by 100.

$27^{103}\ mod\ 100$

$100 can\ be\ written\ as\ 25 * 4$

$27^{103} mod\ 4 = (-1)^{103} mod\ 4 = -1 = 3$

$27^{103} mod\ 25 = 2^{103} mod\ 25 = (2^{10})^{10} * 8 mod\ 25$

$= (1024)^{10} * 8 mod\ 25 = 1 * 8 = 8$